An analytical demonstration of AHP-based MCDM and how it is used in GIS

Understanding Spatial Multi-Criteria Decision Making

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Memphis-in-May NCRST-SEPP Workshop
May 7th, 2009

Topics presented

- **Multi-Criteria:**
  What it is? How does it works? Techniques available
- **Analytical Hierarchy Process**
- **AHP → GIS → Spatial MCDM**
  developing numbers from input rankings
- **Results and future implementations**

What is MCDM ?

Systematic way to select the best available alternatives based on different opinions and conflicting priorities and values.

Haas & Meixner (2006) http://www.boku.ac.at/mi/

Why should we use it?

- **MCDM enables multiple stakeholder preferences to be modeled**
- **MCDM offers improved coordination and collaboration**
- **MCDM can be implemented to integrate spatial information**

How does MCDM works ?

Problem Solving technique

Goal

I-269

Objectives

1) Economy

2) Safety

3) Minimum environmental impact

Factors

1) Desired distance from urban areas

2) Avoid wetlands and forest

3) Stay out (but not far) of ag fields

Criteria

1) D < 1mi → very high

2) 1mi < D < 2mi → high

3) 2mi < D < 3mi → med

4) 3mi < D < 4mi → low

5) 4mi < D < 6mi → med

6) D > 6mi → high

Alternatives

1) B1

2) B2

3) B3

"Problem Solving" techniques

- **SAW** (Simple Additive Weighing)
- **TOPSIS** (Technique for Order Preferences by Similarity to the Ideal Solution)
- **AHP** (Analytical Hierarchy Process)

and more...

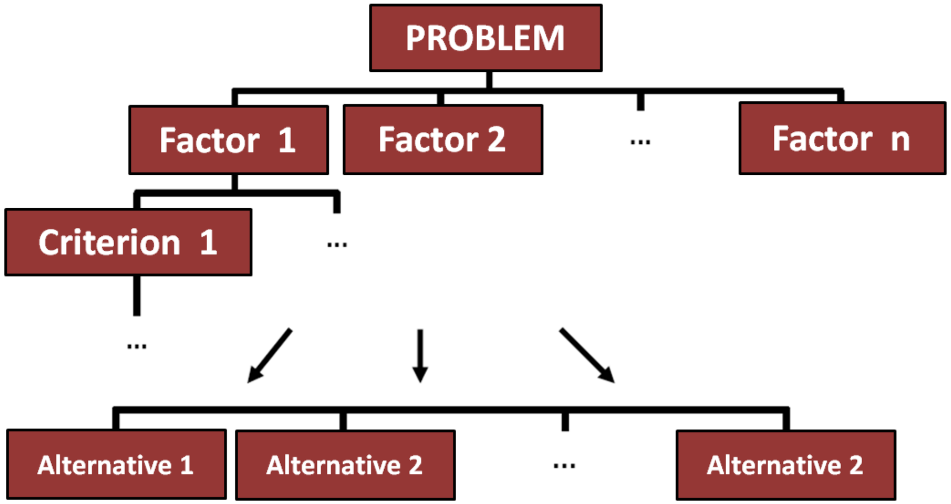
- ELECTRE (Elimination et Choice Translating Reality)
- Bayesian Network Based Framework
- SMART (Simple Multiple Attribute Rating Technique)
- ANP (Analytic Network Process)

Analytic Hierarchy Process - AHP

- **It is a very robust problem solve technique based on pairwise comparisons, developed in early 70's by Dr. Thomas Saaty as a method to help solve conflicts in ecomonic models.**
- **MCDM has been adapted from AHP to assist numerous corporate and govenment decision makers in different fields**
- **Problems are decomposed into a hierarchy of factors and criteria.**

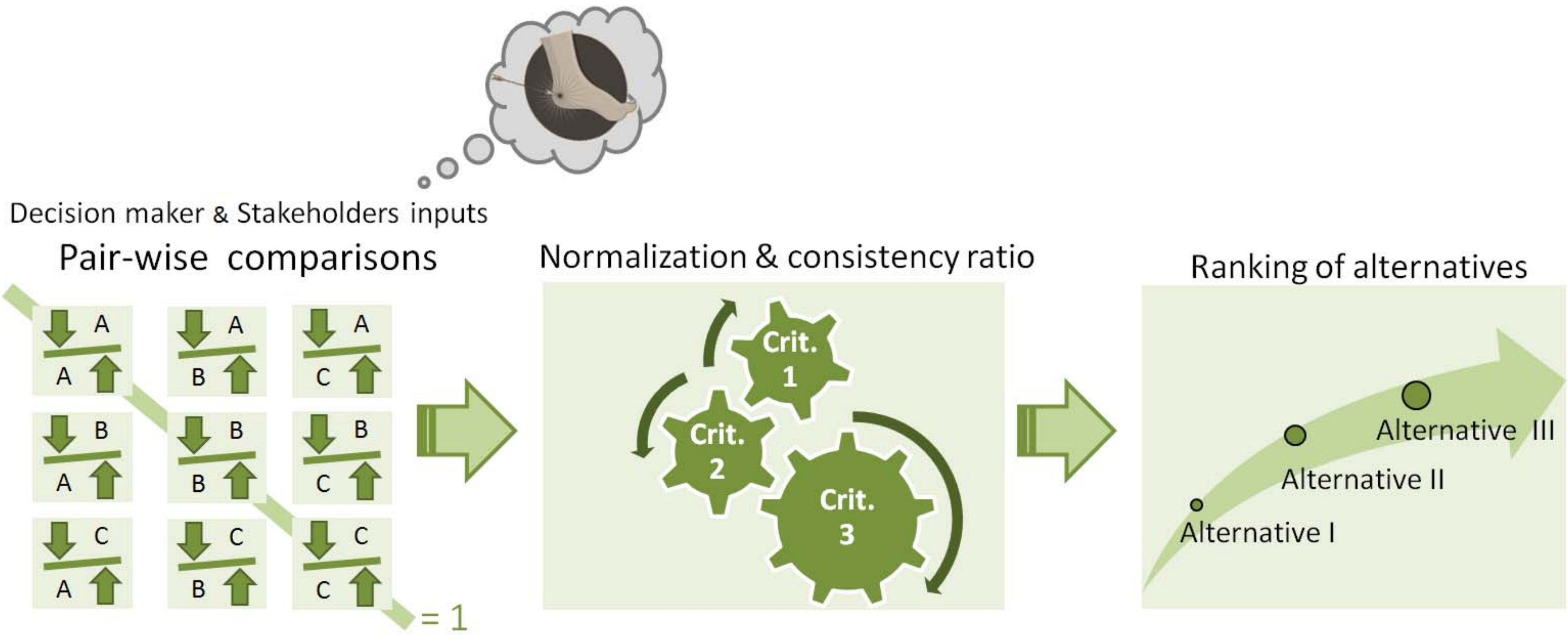
AHP flowchart

- AHP uses a hierarchical structure to solve problems
  Factors and criteria → multi-level



Adapted from Haas & Meixner (2006)

AHP – procedures



AHP – pair-wise comparisons

Pair-wise comparisons should use the Saaty's scale, which ranges from 1 (equal value) to 9 (extreme different)

Pair-wise is applicable for all levels of the AHP process (concurrent factors and concurrent criteria as well)

Intensity of Importance	Definition	Explanation
1	Of equal value	Two requirements are of equal value
3	Slightly more value	Experience slightly favors one requirement over another
5	Essential or strong value	Experience strongly favors one requirement over another
7	Very strong value	A requirement is strongly favored and its dominance is demonstrated in practice
9	Extreme value	The evidence favoring one over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between two adjacent judgments	When compromise is needed
Reciprocals	If requirement one has one of the above numbers assigned to it when compared with requirement second, then second has the reciprocal value when compared with first	

Scale for pair-wise comparison (Saaty 1980)

AHP – normalization and consistency analysis

CRITERIA	urbanized	wetlands	water	slope	Normalized Wt.	
urbanized	**1.0**	3.00	4.50	9.00	urbanized	**0.6000**
wetlands	0.3	**1.00**	1.50	3.00	wetlands	**0.2000**
water	0.2	0.67	**1.00**	2.00	water	**0.1333**
slope	0.1	0.33	0.50	**1.00**	slope	**0.0667**

pair-wise inputs

normalized inputs

Consistency Ratio

Step1 x Step3	Consistency Vector	λ	Consitency Index	Consitency Ratio
2.40	4.00	1.60	-0.80	**-0.2000**
0.80	4.00			PASSED!
0.53	4.00			
0.27	4.00			

Saaty's Inconsistency Indices RI (n=10 Classes)
" if RI < 0.1 ® weights are OK "

n	*RI*
n=2	0.00
n=3	0.58
n=4	0.9
n=5	1.12
n=6	1.24
n=7	1.32
n=8	1.41
n=9	1.46
n=10	1.49

Normalization: " behind the scene"

For a matrix of pair-wise elements:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

1) sum the values in each column of the pair-wise matrix

$$C_{ij} = \sum_{i=1}^{n} C_{ij}$$

2) divide each element in the matrix by its column total to generate a normalized pair-wise matrix

$$X_{ij} = \frac{C_{ij}}{\sum_{i=1}^{n} C_{ij}} \qquad \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix}$$

3) divide the sum of the normalized column of matrix by the number of criteria used (n) to generate weighted matrix

$$W_{ij} = \frac{\sum_{j=1}^{n} X_{ij}}{n} \qquad \begin{bmatrix} W_{11} \\ W_{12} \\ W_{13} \end{bmatrix}$$

Consistency analysis: “behind the scene”

Consistency Vector is calculated by multiplying the pair-wise matrix by the weights vector

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} * \begin{bmatrix} W_{11} \\ W_{21} \\ W_{31} \end{bmatrix} = \begin{bmatrix} Cv_{11} \\ Cv_{21} \\ Cv_{31} \end{bmatrix}$$

Then it is is accomplished by dividing the weighted sum vector with criterion weight

$$Cv_{11} = \frac{1}{W_{11}}[C_{11}W_{11} + \quad C_{12}W_{21} \quad C_{13}W_{31}]$$

$$Cv_{21} = \frac{1}{W_{21}}[C_{21}W_{11} + \quad C_{22}W_{21} \quad C_{23}W_{31}]$$

$$Cv_{31} = \frac{1}{W_{31}}[C_{31}W_{11} + \quad C_{32}W_{21} \quad C_{33}W_{31}]$$

λ is calculated by averaging the value of the Consistency Vector

$$\lambda = \sum_{i=1}^{n} Cv_{ij}$$

$$CI = \frac{\lambda - n}{n-1}$$

CI measures the deviation

$$C_r = \frac{CI}{RI}$$

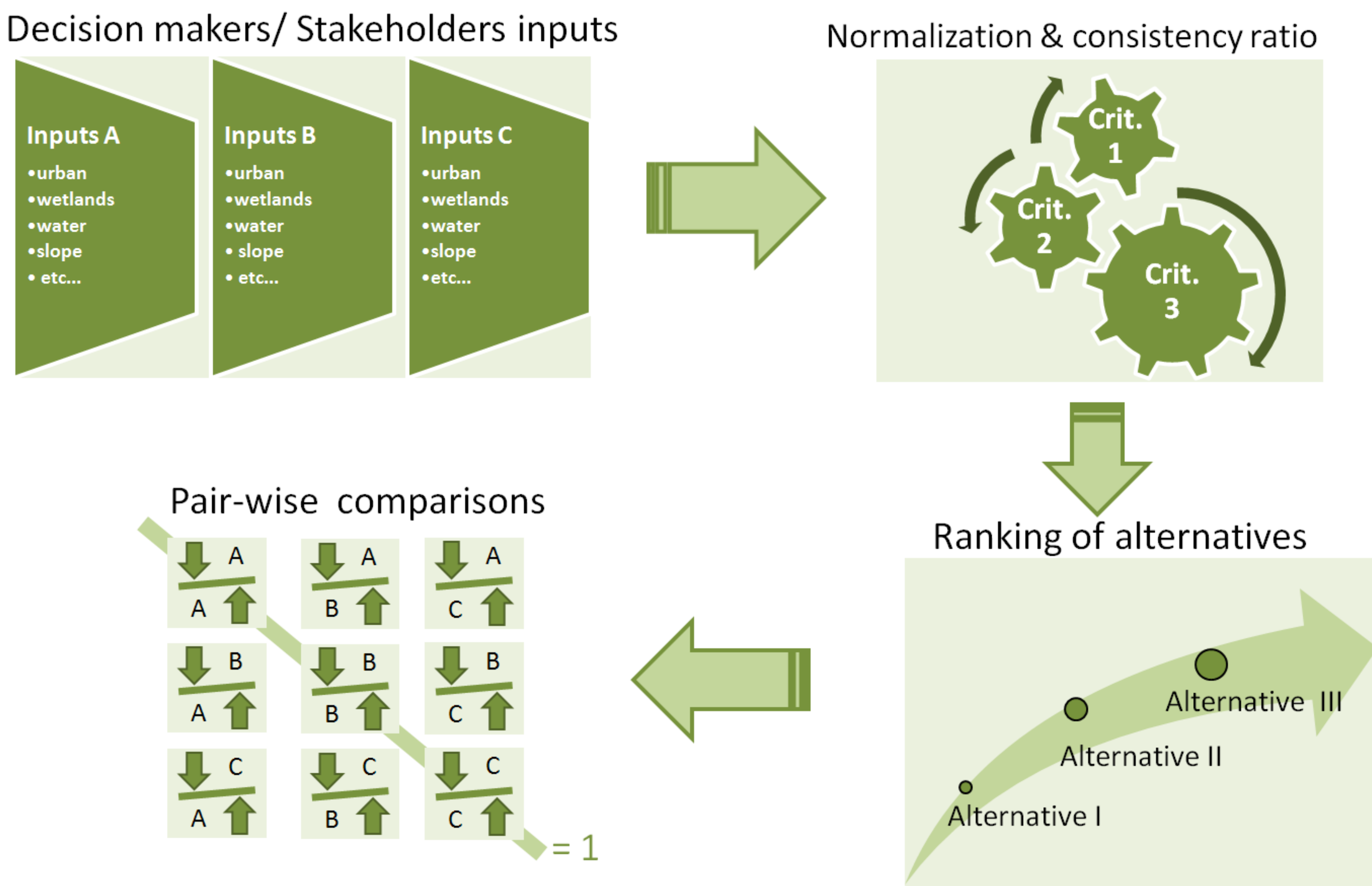
$C_r < 0.10$

RANDOM INCONSISTENCY INDICES (RI) FOR $N = 10$

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.9	1.12	1.24	1.32	1.41	1.46	1.49

Source: Satty (1980).

Real world needs: <u>ranking</u> instead pair-wise inputs



AHP + GIS = Spatial MCDM

1) Selecting factors
   "positive/negative impact for environment cost, etc."

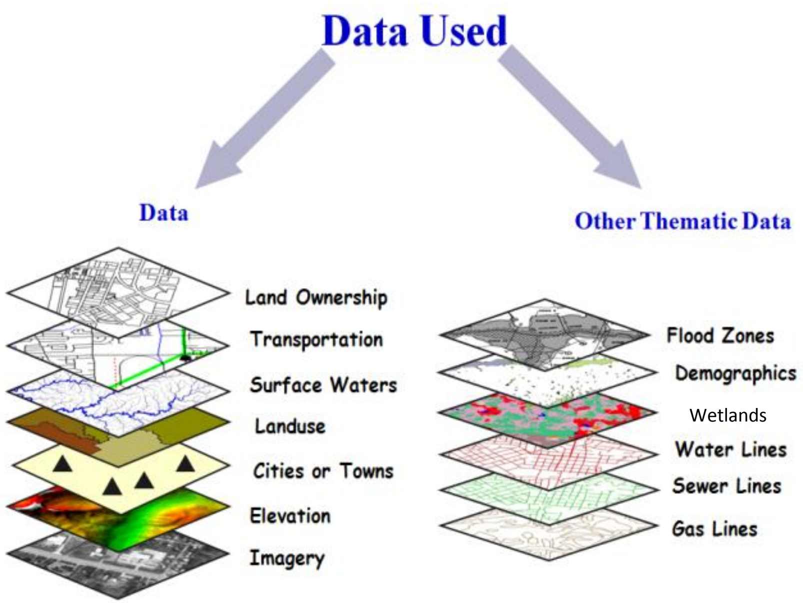
2) Ranking criteria (single scenarios)
   "quantifying degree of influence → ex: distance from Wetlands"

3) Ranking factors (combined scenarios)
   "quantifying importance of factors → ex: Wetlands X Agriculture"

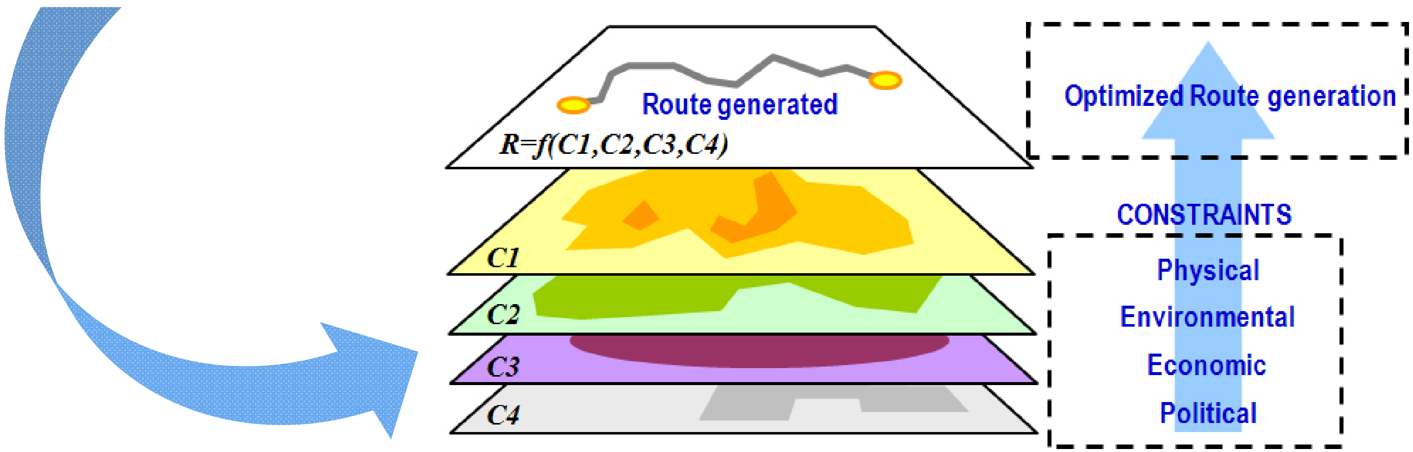
4) From ranking to weights
   "mathematical approach based on pair-wise comparisons"

5) Least-Cost Path
   " GIS approach with map algebra "

AHP + GIS = Spatial MCDM

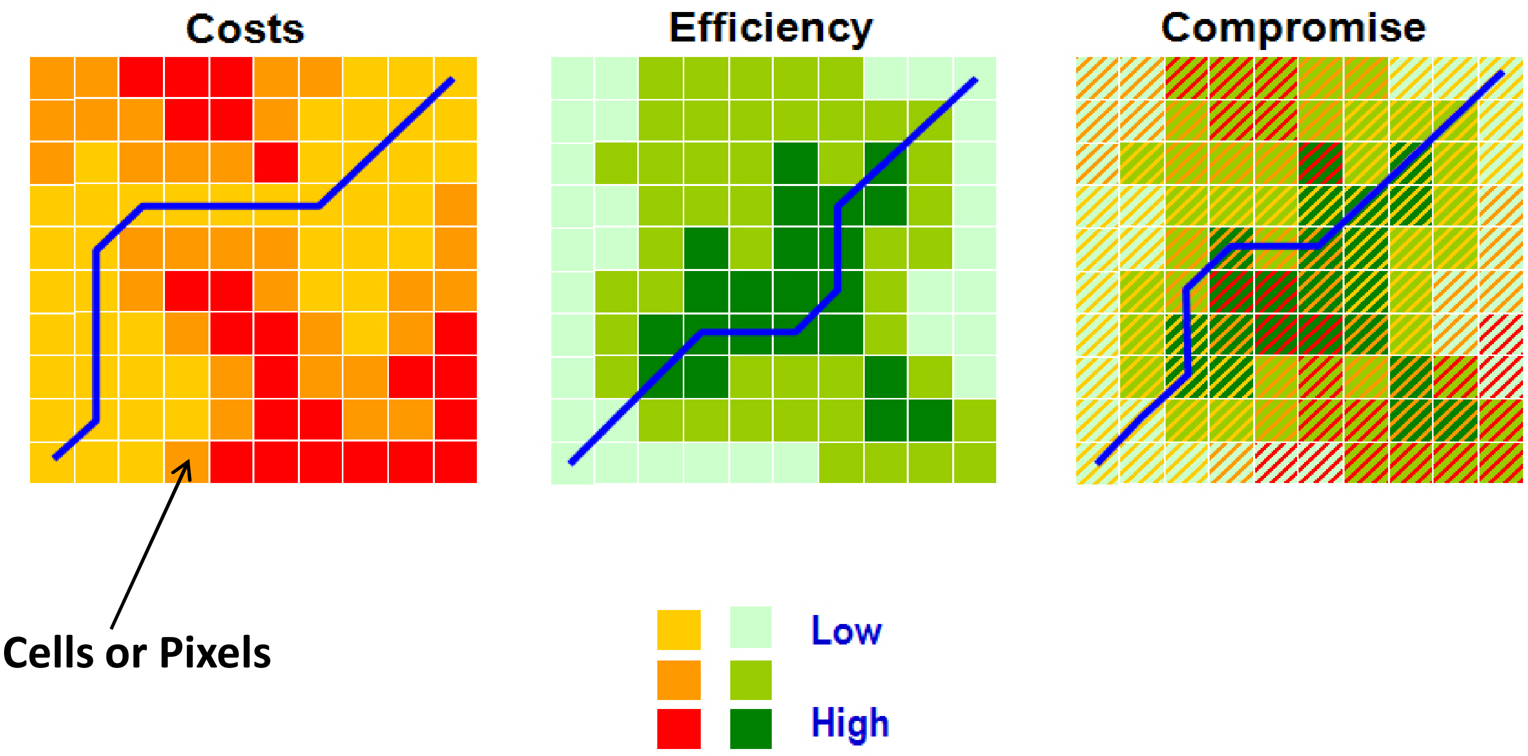


- Weights represent the resistance, friction or difficulty in crossing the cell which is expressed as cost
- Creation of accumulated-cost-surface grid from a cost-of-passage where friction values are stored
- Tracing a line of least cost from the accumulated-cost-surface (Douglas 1994)



AHP + GIS = Spatial MCDM

Data → raster format (digital image)

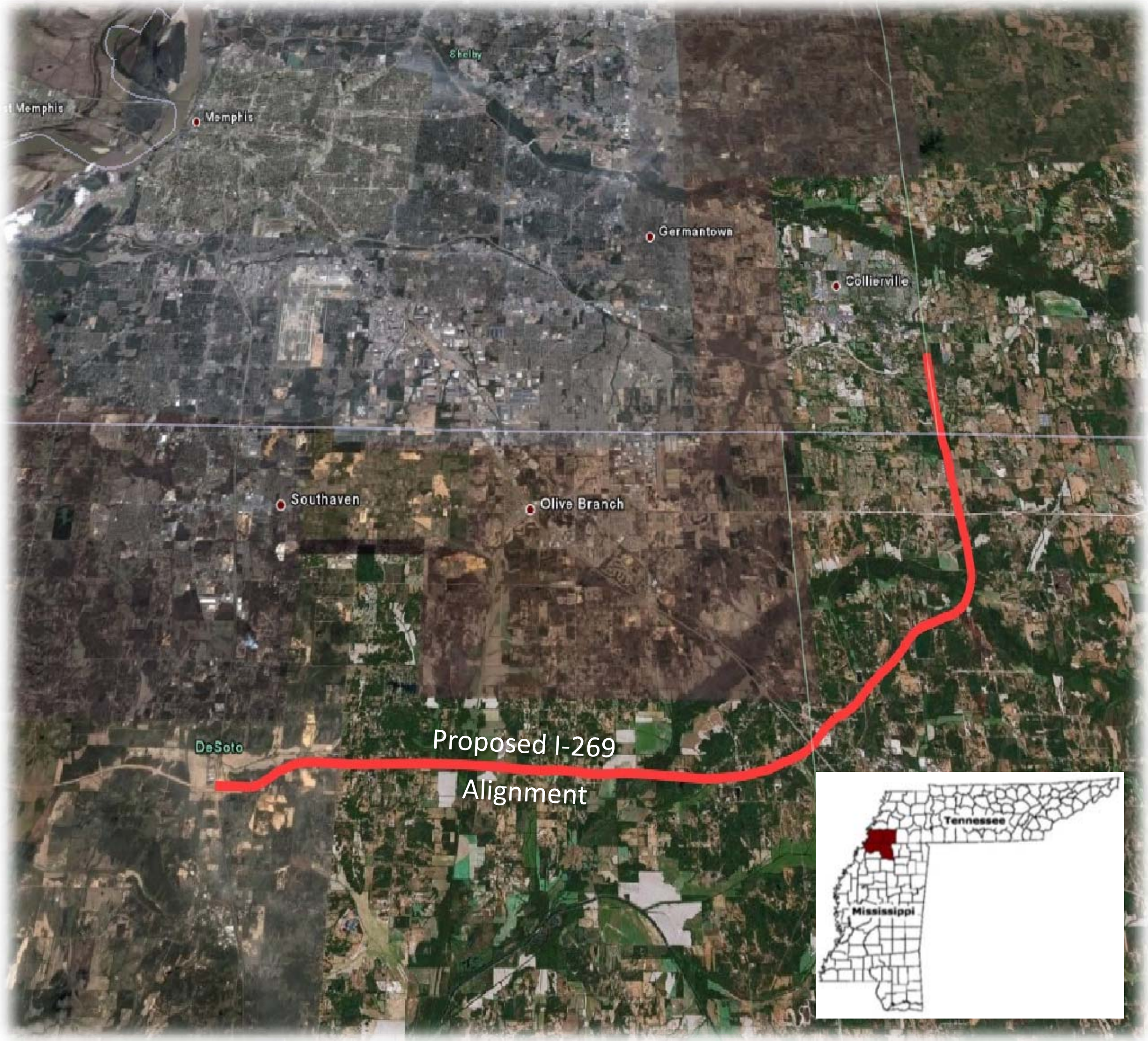


Spatial MCDM: case application

Study Area

The testbed used is a part of the I-269

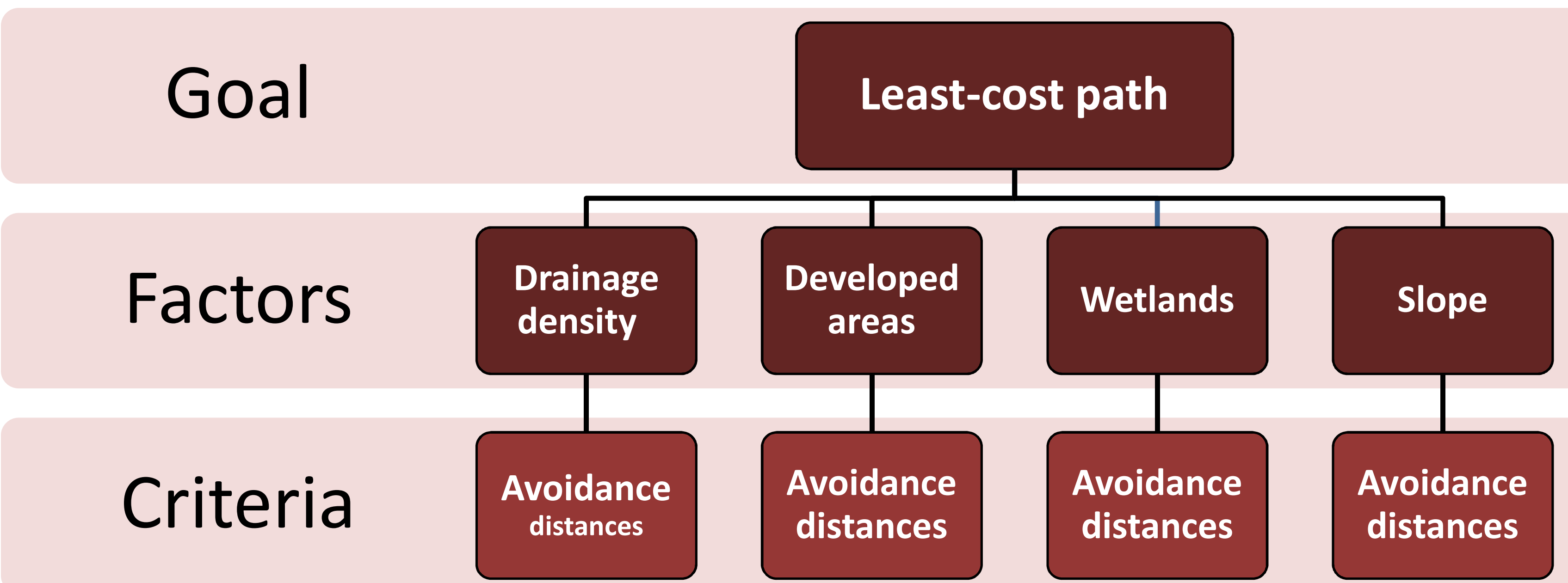
Around 30-mile corridor that connects Hernando-MS to Collierville-TN



Spatial MCDM: case application (hypothetical values)

Four factors:

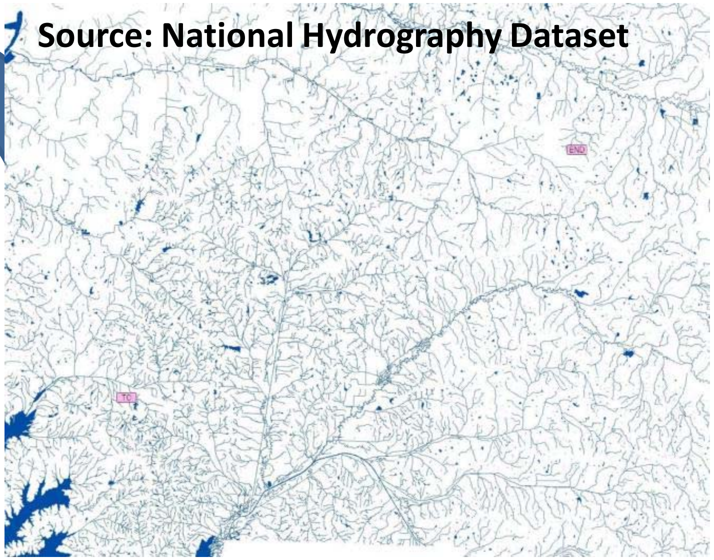
- **Drainage density (waterbodies + streams)**
- **Developed areas**
- **Wetlands**
- **Slope**

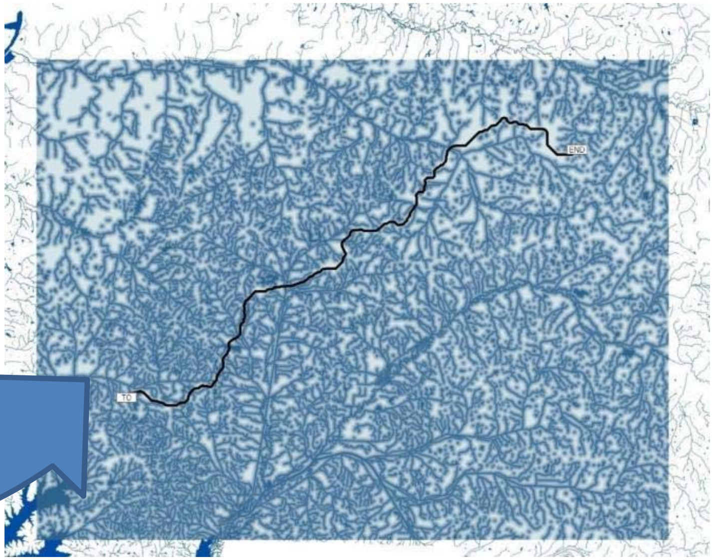


Hydrography

Distance from Water	Ranking
0 – 50 m	3
50 – 300 m	2
> 300 m	1

Source: National Hydrography Dataset





Criteria inputs:

3 = close
2 = medium
1 = far

Computing weights

Step – I				Step – II				Step – III	
Classes	DD1	DD2	DD3	Classes	Cl1	Cl2	Cl3	/ n	Std.Wt
DD1	1	2	3	Cl1	0.5455	0.5714	0.5	0.5455+0.5714+0.5	0.5389
DD2	1/2	1	2	Cl2	0.2727	0.2857	0.3333	0.2727+0.2857+0.3333	0.2972
DD3	1/3	1/2	1	Cl3	0.1818	0.1428	0.1666	0.1818+0.1428+0.1666	0.1637
Totals	1.833	3.5	6.0	..					

Consistency ratio analysis:

Step – I

Classes	DD1	DD2	DD3		Std.Wts
DD1	1	2	3		0.5389
DD2	1/2	1	2	*	0.2972
DD3	1/3	1/2	1		0.1637

= Cv: 1.6244, 0.89405, 0.4919 →

Step – II

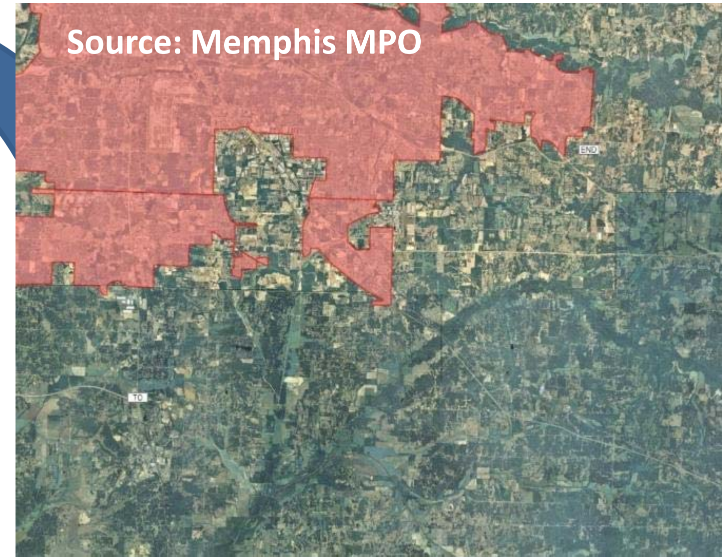
Cv_1 1.6244 / 0.5389 = 3.0142
Cv_2 0.89405 / 0.2972 = 3.0082
Cv_3 0.4919 / 0.1637 = 3.0048
$\lambda = 3.009$ Average – this – column

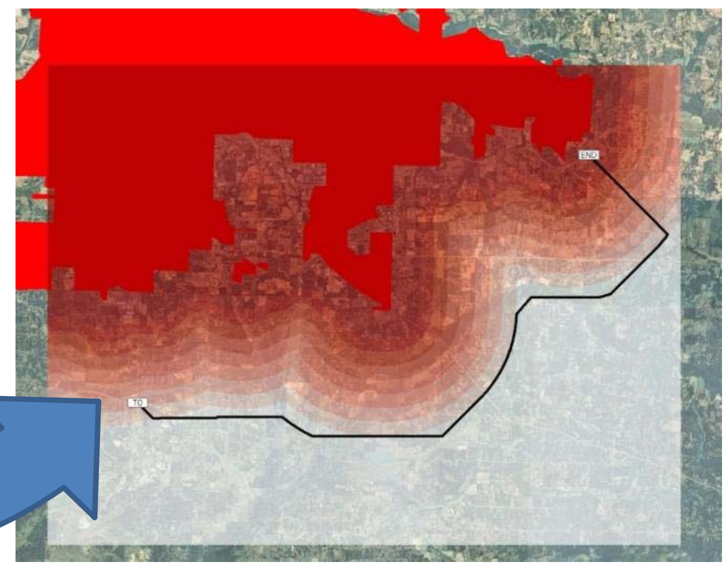
$$\rightarrow CI = \frac{3.009 - 3.0}{2} = 0.0045 \rightarrow C_r = \frac{0.0045}{0.58} = 0.0077$$

$C_r < 0.1$

Distance from MPO Urbanized Limits

Distance from MPO	Ranking
0 – 2 Km	5
2 – 4 Km	4
4 – 6 Km	3
6 – 8 Km	2
> 8 Km	1





Criteria inputs:

5 = inner city
4 = close
3 = medium
2 = far
1 = so far

Computing weights

Step–I						*Step–II*					*Step–III*	
Classes	UD1	UD2	UD3	UD4	UD5	UD1	UD2	UD3	UD4	UD5	/ *n*	*StdWeigth*
UD1	1	2	3	4	5	0.438	0.439	0.439	0.381	0.333	0.438+0.489+0.439+0.381+0.333	0.4162
UD2	0.5	1	2	3	4	0.219	0.244	0.292	0.285	0.266	0.219+0.244+0.292+0.285+0.266	0.2618
UD3	0.333	0.5	1	2	3	0.146	0.122	0.146	0.190	0.200	0.146+0.122+0.146+0.190+0.200	0.1611
UD4	0.25	0.333	0.5	1	2	0.109	0.081	0.073	0.095	0.133	0.109+0.081+0.073+0.095+0.133	0.0986
UD5	0.2	0.25	0.333	0.5	1	0.087	0.061	0.048	0.047	0.066	0.087+0.061+0.048+0.047+0.066	0.0624
Totals	2.2833	4.0833	6.8333	10.5	15							

Consistency ratio analysis:

$C_r < 0.1$

Step – I

Classes	*UD1*	*UD2*	*UD3*	*UD4*	*UD5*		*Std.Wts*		*Cv*
UD1	1	2	3	4	5		0.4162		2.1291
UD2	0.5	1	2	3	4		0.2618		1.3372
UD3	0.3333	0.5	1	2	3	*	0.1611	=	0.8150
UD4	0.25	0.3333	0.5	1	2		0.0986		0.4952
UD5	0.2	0.25	0.3333	0.5	1		0.0624		0.3140

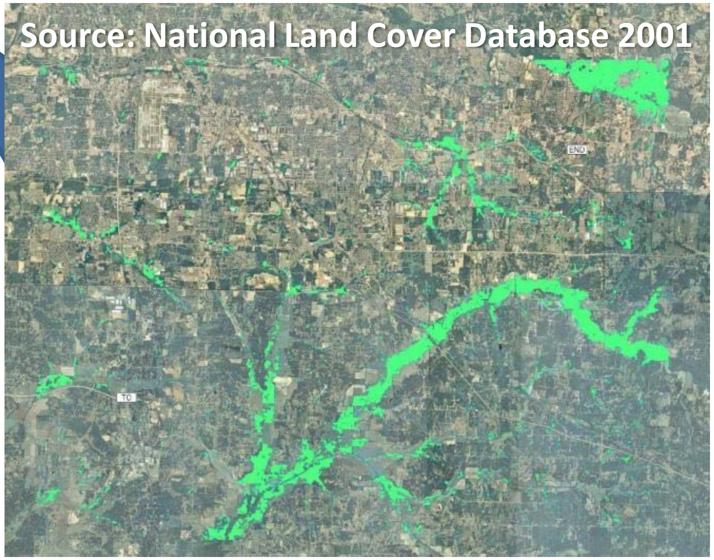
→

Step – II

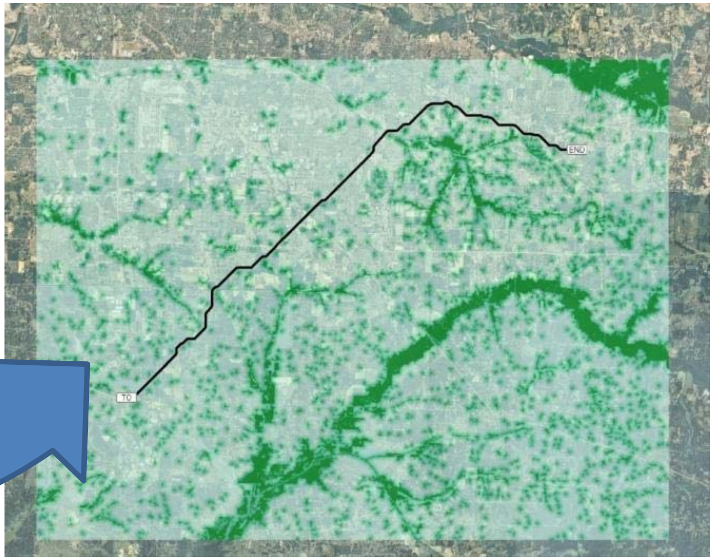
Cv_1	$2.1291 / 0.4162 = 5.11$
Cv_2	$1.3372 / 0.2618 = 5.1080$
Cv_3	$0.8150 / 0.2618 = 5.0603$
Cv_4	$0.4952 / 0.0986 = 5.0234$
Cv_5	$0.3140 / 0.0624 = 5.0345$
$\lambda = 2.5342$	*Average – this – last – column*

$$\rightarrow CI = \frac{2.5342 - 5.0}{4} = -0.6165 \rightarrow C_r = \frac{-0.6165}{1.12} = -0.1233$$

Wetlands



Distance from Wetlands	Ranking
0 – 50 m	3
50 – 200 m	2
> 200 m	1



Criteria inputs:

3 = close
2 = medium
1 = far

Computing weights

	Step – I				Step – II			Step – III	
Classes	Wl1	Wl2	Wl3	Classes	Cl1	Cl2	Cl3	/ n	Std.Wt
Wl1	1	2	3	Wl1	0.5455	0.5714	0.5	0.5455 + 0.5714 + 0.5	0.5389
Wl2	1/2	1	2	Wl2	0.2727	0.2857	0.3333	0.2727 + 0.2857 + 0.3333	0.2972
Wl3	1/3	1/2	1	Wl3	0.1818	0.1428	0.1666	0.1818 + 0.1428 + 0.1666	0.1637
Totals	1.833	3.5	6.0	..					

Consistency ratio analysis:

Step – I					
Classes	Wl1	Wl2	Wl3		Std.Wts
Wl1	1	2	3		0.5389
Wl2	1/2	1	2	*	0.2972
Wl3	1/3	1/2	1		0.1637

=

Cv
1.6244
0.89405
0.4919

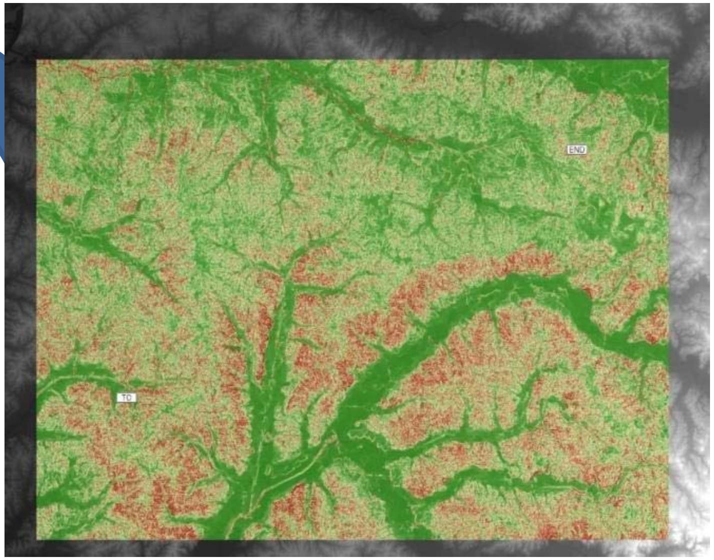
→

Step – II	
Cv_1	1.6244 / 0.5389 = 3.0142
Cv_2	0.89405 / 0.2972 = 3.0082
Cv_3	0.4919 / 0.1637 = 3.0048
$\lambda = 3.009$	Average – this – column

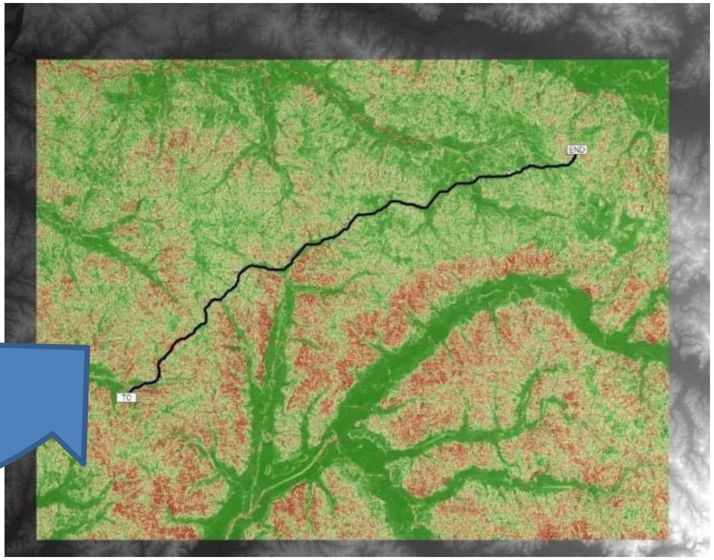
$$\rightarrow CI = \frac{3.009 - 3.0}{2} = 0.0045 \rightarrow C_r = \frac{0.0045}{0.58} = 0.0077$$

$C_r < 0.1$

Topography



Slope	Ranking
< 5%	1
5-20%	3
> 20%	6



Criteria inputs:

6 = Rugged
3 = medium
1 = flat

Computing weights

	Step – I				Step – II			Step – III	
Classes	SC1	SC2	SC3	Classes	Cl1	Cl2	Cl3	/ n	Std.Wt
SC1	1	4	6	LD1	0.7092	0.7504	0.6	0.7092 + 0.7504 + 0.6	0.6865
SC2	1/4	1	3	LD2	0.1773	0.1876	0.3	0.1773 + 0.1876 + 0.3	0.2216
SC3	1/6	1/3	1	LD3	0.1182	0.0625	0.1	0.1182 + 0.0625 + 0.1	0.0935
Totals	1.41	5.33	10.0	..					

Consistency ratio analysis:

$C_r < 0.1$

Step – I					
Classes	SC1	SC2	SC3		Std.Wts
SC1	1	4	6		0.6865
SC2	1/4	1	3	*	0.2216
SC3	1/6	1/3	1		0.0935

$$= \begin{bmatrix} Cv \\ 2.1339 \\ 0.6737 \\ 0.2816 \end{bmatrix} \rightarrow$$

Step – II	
Cv_1	$2.1339 / 0.6865 = 3.1083$
Cv_2	$0.6737 / 0.2216 = 3.0401$
Cv_3	$0.2816 / 0.0935 = 3.0117$
$\lambda = 3.0532$	Average – this – column

$$\rightarrow CI = \frac{3.0532 - 3.0}{2} = 0.0266 \rightarrow C_r = \frac{0.0266}{0.58} = 0.045$$

Combining multiple scenarios (hypothetical values)

7 = develope area
4 = drainage density
1 = slope
3 = wetlands

MCDM

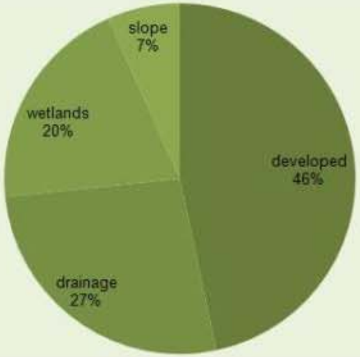
ranking	CRITERIA	Step 1: developed 7	drainage 4	wetlands 3	slope 1	Step 2				Step 3: Normalized Wt.	
7	developed	1.0	1.75	2.33	7.00	0.467	0.467	0.467	0.467	developed	0.4667
4	drainage	0.6	1.00	1.33	4.00	0.267	0.267	0.267	0.267	drainage	0.2667
3	wetlands	0.4	0.8	1.00	3.00	0.200	0.200	0.200	0.200	wetlands	0.2000
1	slope	0.1	0.3	0.33	1.00	0.067	0.067	0.067	0.067	slope	0.0667
	4	2.1	3.75	5.00	15.00	1.0	1.0	1.0	1.0		1.0000

Consistency Ratio

Step1 x Step3	Consistency Vector	λ	Consitency Index	Consitency Ratio
1.87	4.00	1.60	-0.80	-0.2000
1.07	4.00			PASSED!
0.80	4.00			
0.27	4.00			

Saaty's Inconsistency Indices RI
(n=10 Classes)
" if RI < 0.1 ® weights are OK "

n	RI
n=2	0.00
n=3	0.58
n=4	0.9
n=5	1.12
n=6	1.24
n=7	1.32
n=8	1.41
n=9	1.46
n=10	1.49



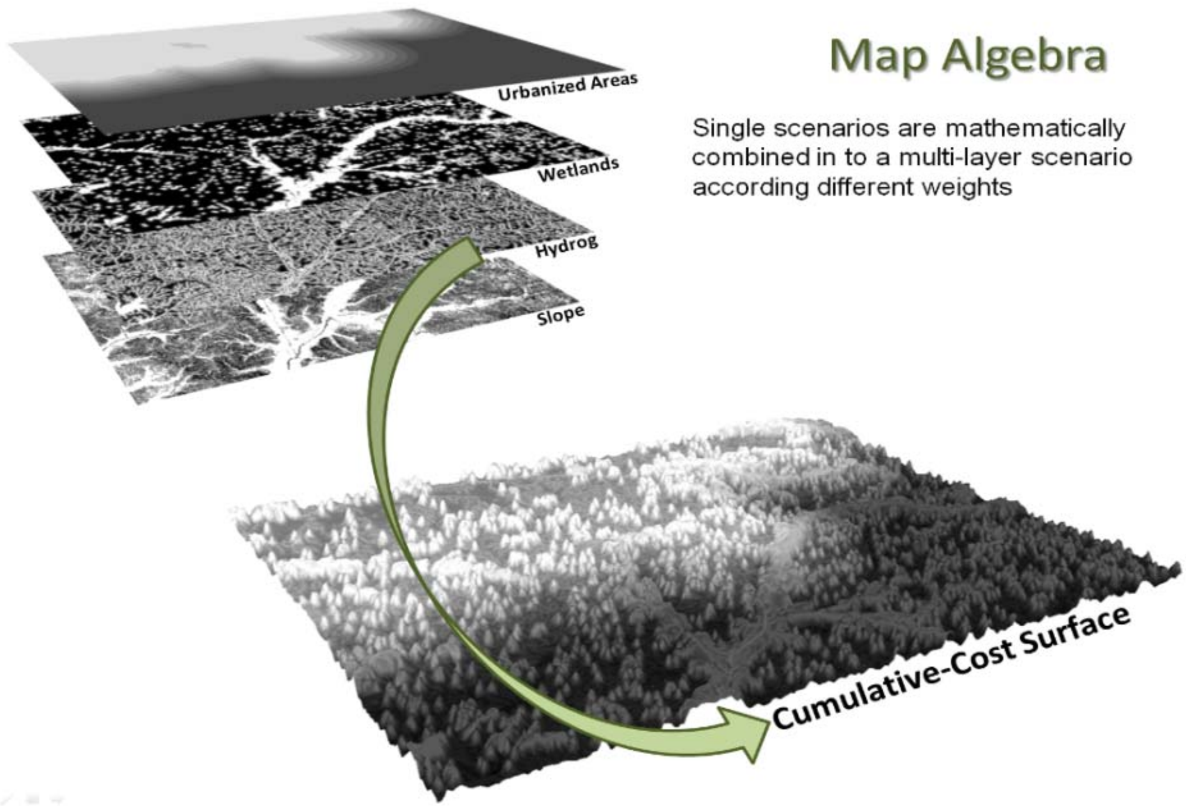
Combining multiple scenarios

Factor	Rank	Weight
UD	7	0.4667
DD	4	0.2667
WL	3	0.2000
SC	1	0.0667

$$0.4667*(UD) + 0.2667*(DD) + 0.2*(WL) + 0.0667*(SC)$$

In multiple layer cases, assigned numerical values that provide relative weights are also normalized.

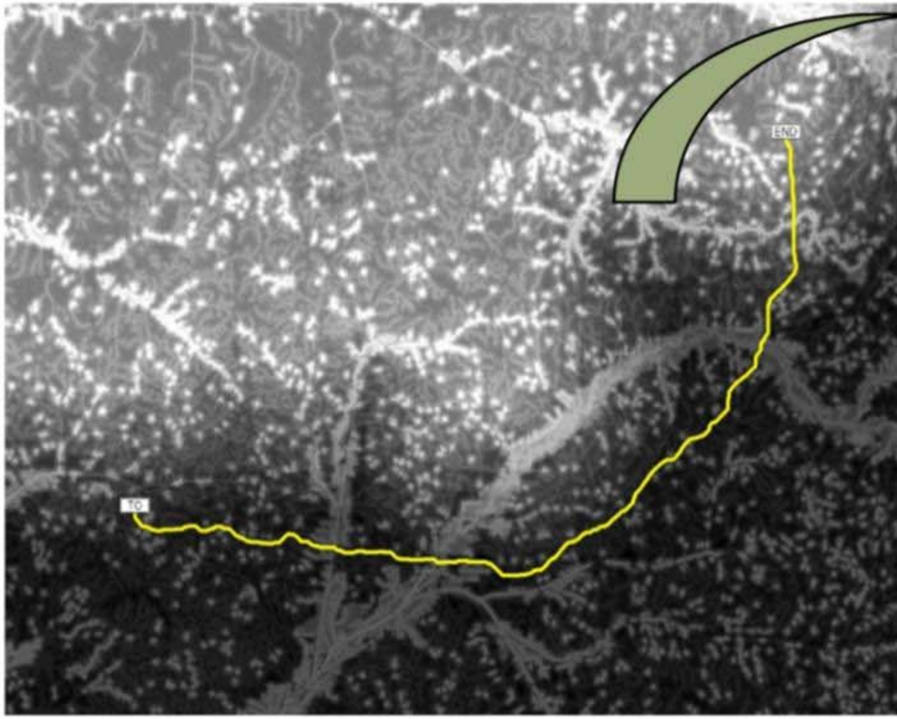
In this approach, each stakeholder may select weights that match their personal and professional perspective and values to create a unique cost surface and cost path!



Map Algebra

Single scenarios are mathematically combined in to a multi-layer scenario according different weights

Least-Cost Path



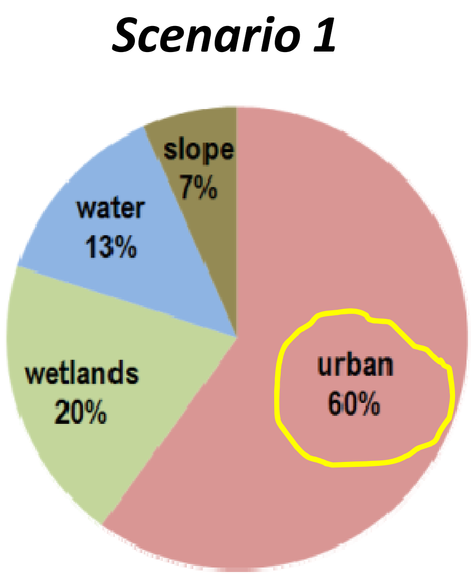
Cumulative cost surface and the least-cost path

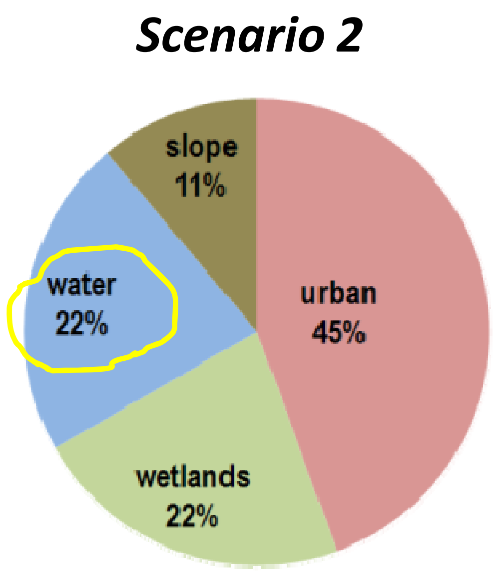
Least-cost path visualized using Google Earth

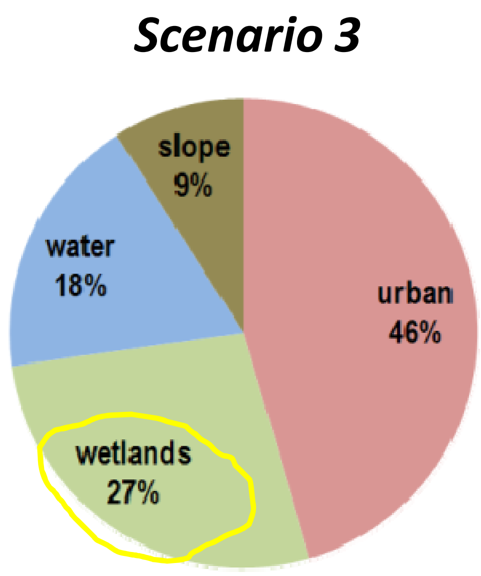
SENSITIVE ANALYSIS USING MCDM

Putting together different scenarios

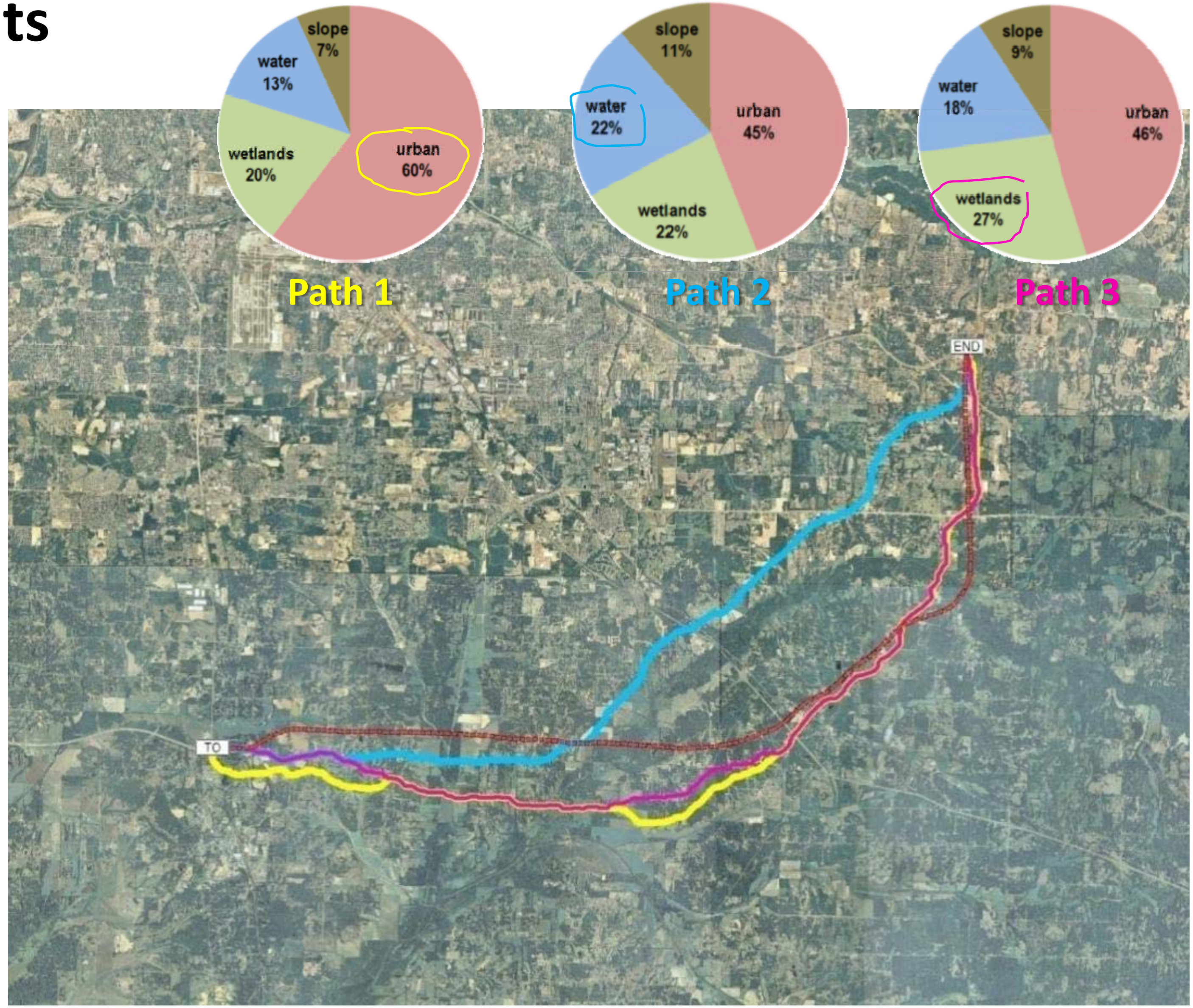
INPUT → rankings
OUTPUT → least-cost path







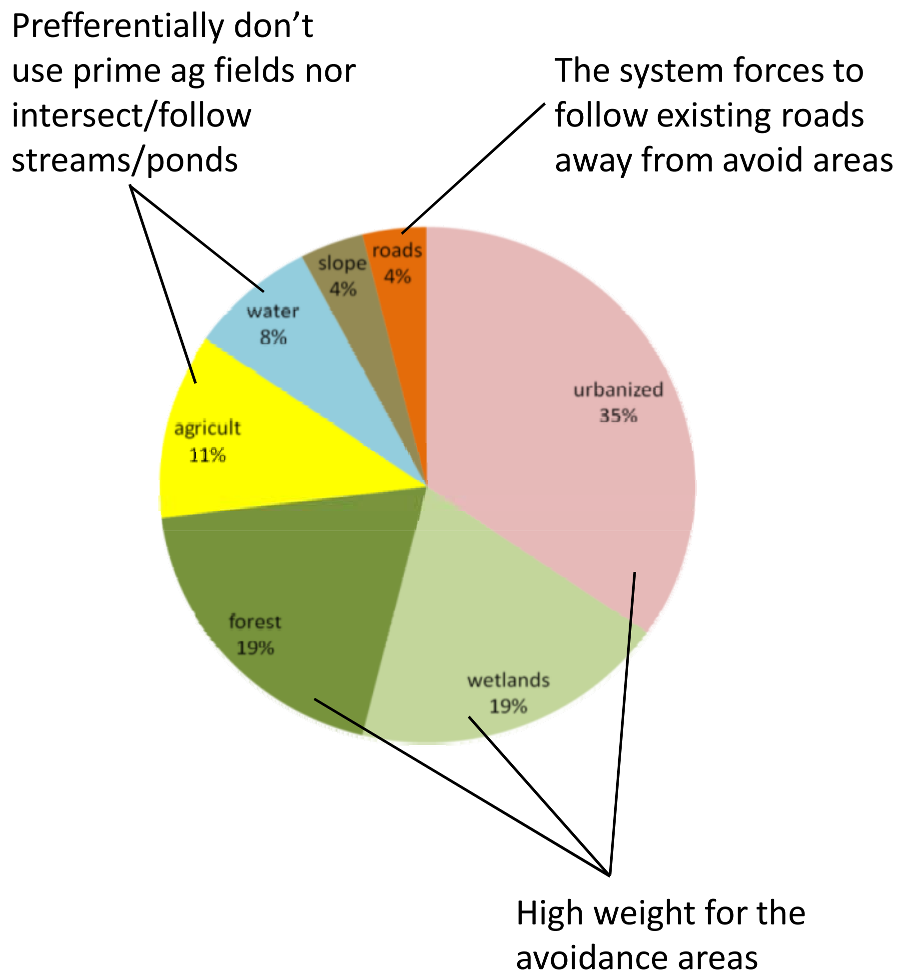
Results


slope 7%
water 13%
wetlands 20%
urban 60%
Path 1
slope 11%
water 22%
urban 45%
wetlands 22%
Path 2
slope 9%
water 18%
urban 46%
wetlands 27%
Path 3
END
TO

ADDING MORE FACTORS

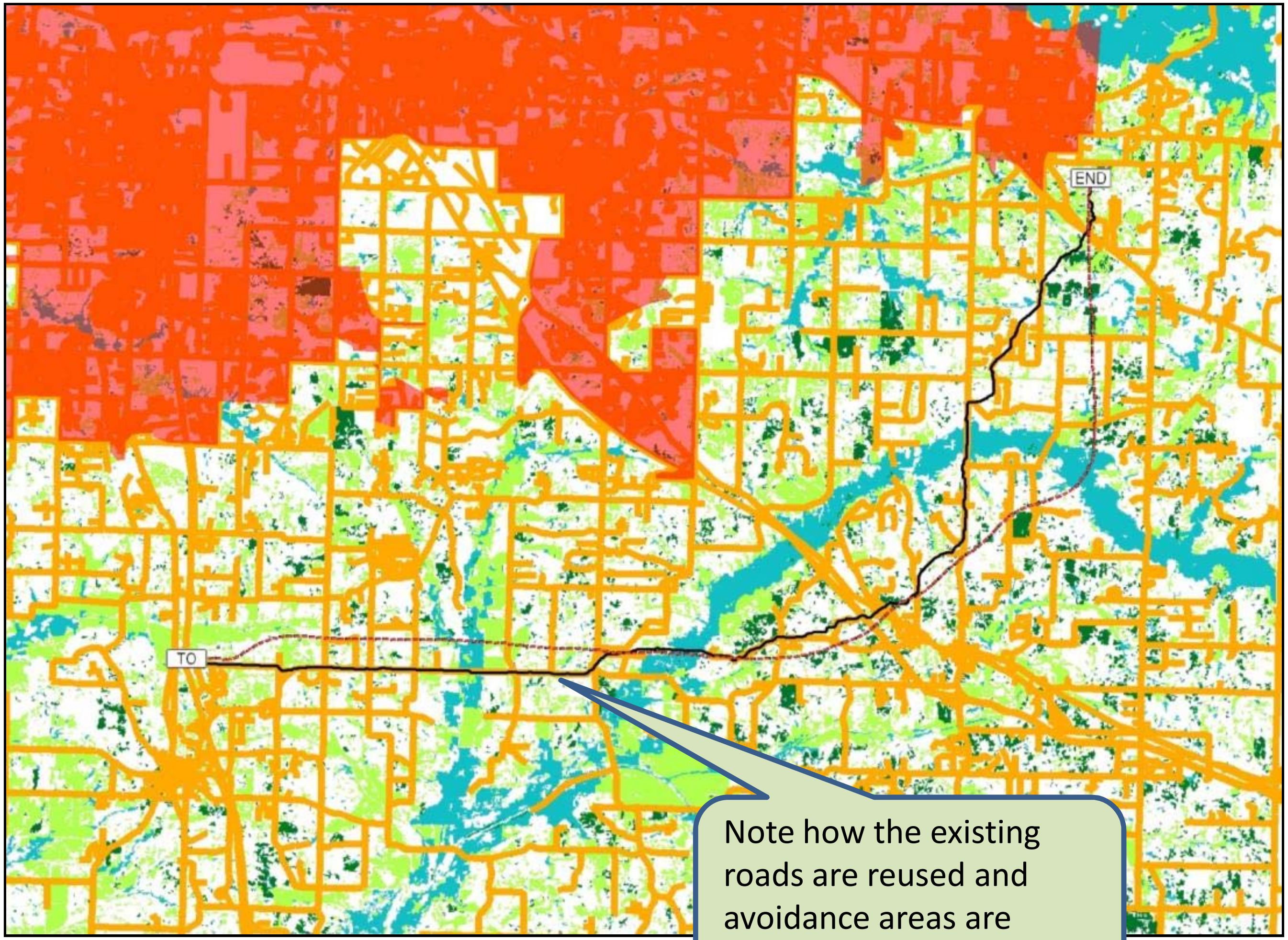
Increasing the complexity of the analysis

FACTOR	Ranking
MPO urban limits	**9** *criteria: dist from 0 – 7 Km*
Wetlands	**5** *avoidance*
Forest	**5** *avoidance*
Agriculture	**3** *High cost*
Hydrography	**2** *criteria: dist from 0 – 300 m*
Roads	**1** *Reuse existing roads*
Slope	**1** *0- 20% , >20%*



Experimenting Aditional Factor and Scenarios

Overlaying : MPO urban + Ag + wetlands + forest + existing roads



Note how the existing roads are reused and avoidance areas are considered

MCDM Research Results

- **Feb 2009 - MSU Transportation Workshop**
  Poster presentation: NOBREGA et al. Environmental sensitive corridor planning using MCDM
- **March 2009 – ASPRS Annual Conference**
  Paper/Oral presentation: SADASIVUNI et al. A transportation corridor case study for multi-criteria decision analysis.
- **April 2009 – Management of Environmental Qualify International Journal**
  Journal paper (submitted): NOBREGA et al. Bridging decision making process and environmental needs in transportation corridor planning
- **Journal papers in progress:**
  MCDM and non-traditional remote sensing data inputs (in collaboration with MTRI)

  An innovative MCDM approach for corridor planning based on integrated multi-scale data and AHP method

That's it!

Next presentation: application & results